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$B_{s,d} \to \ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ in SUSY models with non-minimal sources of flavour mixing

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Abstract: We present a general analysis of $B_{s,d} \to \ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ decays in supersymmetric models with non-minimal sources of flavour mixing. In spite of the existing constraints on off-diagonal squark mass terms, these modes could still receive sizeable corrections, mainly because of Higgs-mediated FCNCs arising at large $\tan\beta$. The severe limits on scenarios with large $\tan\beta$ and non-negligible $\tilde{d}^i_{R(L)}$–$\tilde{d}^j_{R(L)}$ mixing imposed by the present experimental bounds on these modes and $\Delta B = 2$ observables are discussed in detail. In particular, we show that scalar-current contributions to $K_L \to \ell^+\ell^-$ and B–$\bar{B}$ mixing set non-trivial constraints on the possibility that $B_s \to \ell^+\ell^-$ and $B_d \to \ell^+\ell^-$ receive large corrections.

Keywords: Supersymmetric Standard Model, Rare Decays, B-Physics.

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Contents

1. Introduction

The rare dilepton decays $B_{s,d} \to \ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ are ideal candidates to deeply investigate the dynamics of quark-flavour mixing. On the one side, they are highly sensitive to physics beyond the Standard Model (SM), like all processes mediated by flavour-changing neutral-current (FCNC) amplitudes. On the other side, the helicity suppression of vector contributions gives to these modes a unique sensitivity to scalar currents, which are typically negligible in other rare decays.

In this paper we analyse the rare dilepton decays in supersymmetric (SUSY) models with flavour non-universal soft-breaking terms. As is well known, this class of models is strongly constrained by currently available data on $\Delta F = 2$ and $\Delta F = 1$ amplitudes (see e.g. ref. [1]). However, a large fraction of the parameter space still remains to be explored. The only conclusion that can be drawn at present is that the possible new sources of flavour mixing, if any, must have a rather non-trivial structure. For instance, as emphasized recently in ref. [2], the possibility of a large $\tilde{b}_R$–$\tilde{s}_R$ mixing, motivated by SO(10) unification and neutrino data, is still open.

Supersymmetric contributions to rare FCNC decays have been widely discussed in the literature. However, most of the analyses are focused on scenarios with $\tan\beta = O(1)$. As we shall show, if $\tan\beta = O(1)$ there is little to learn from the dilepton modes, which cannot be extracted elsewhere. On the other hand, things change substantially at large $\tan\beta$, because of the scalar FCNCs induced by the effective coupling of H_U^* to down-type quarks (the so-called non-holomorphic terms) [3, 4].

As discussed in refs. [3]–[8], at large $\tan\beta$ the rates of $B_{s,d} \to \ell^+\ell^-$ decays could be substantially enhanced with respect to the SM even without new sources of flavour mixing. As pointed out in [4], the same conclusion does not hold for $K_L \to \ell^+\ell^-$. In this paper we analyse how the situation is modified by non-minimal flavour structures. Interestingly enough, in this case also $K_L \to \ell^+\ell^-$ play an important role, setting stringent bounds on off-diagonal squark mass terms.

The issue of SUSY contributions to $B_{s,d} \to \ell^+\ell^-$ decays in the presence of non-universal squark mass terms has already been addressed in refs. [6, 7]. Here we extend these works by discussing all possible $\tan\beta$-enhanced terms, both those arising at higher orders and those not related to scalar FCNCs. We also clarify the issue of the diagonalization of the quark mass matrix in the presence of non-universal non-holomorphic terms. Last but not least, in our analysis we take into account the constraints from scalar-current contributions to $K_L \to \ell^+\ell^-$ and B–$\bar{B}$ mixing, which have not been discussed before.

2. Generalities

The part of the $\Delta F = 1$ effective hamiltonian relevant to $P[\bar{d}^i d^j] \to l^+l^-$ decays is

$$\mathcal{H}^{\rm eff} = -\frac{G_F^2 M_W^2}{\pi^2}\left[C_A \mathcal{O}_A + C'_A \mathcal{O}'_A + C_S \mathcal{O}_S + C_P \mathcal{O}_P + C'_S \mathcal{O}'_S + C'_P \mathcal{O}'_P\right] + \text{h.c.}, \tag{2.1}$$

where

$$\begin{aligned}
\mathcal{O}_A &= \bar{d}^i_L \gamma^\mu d^j_L \bar{\ell}\gamma_\mu\gamma_5\ell\,, & \mathcal{O}'_A &= \bar{d}^i_R \gamma^\mu d^j_R \bar{\ell}\gamma_\mu\gamma_5\ell\,,\\
\mathcal{O}_S &= \bar{d}^i_R d^j_L \bar{\ell}\ell\,, & \mathcal{O}'_S &= \bar{d}^i_L d^j_R \bar{\ell}\ell\,,\\
\mathcal{O}_P &= \bar{d}^i_R d^j_L \bar{\ell}\gamma_5\ell\,, & \mathcal{O}'_P &= \bar{d}^i_L d^j_R \bar{\ell}\gamma_5\ell\,.
\end{aligned} \tag{2.2}$$

According to this normalization of the effective operators, writing hadronic matrix elements of axial and pseudoscalar currents as

$$\langle 0|\bar{q}^i\gamma_\mu\gamma_5 q^j|P(p)\rangle = i p_\mu f_P\,, \qquad \langle 0|\bar{q}^i\gamma_5 q^j|P(p)\rangle = -i f_P \frac{M_P^2}{(m_i+m_j)}\,, \tag{2.3}$$

the most general expression of the short-distance (SD) contribution to $P[\bar{d}^i d^j] \to l^+l^-$ branching ratios reads

$$\begin{aligned}
\mathcal{B}_{\rm SD}(P\to\ell^+\ell^-) = {}& \frac{G_F^4 M_W^4}{8\pi^5} f_P^2 M_P \tau_P m_\ell^2 \left(1-\frac{4m_\ell^2}{M_P^2}\right)^{1/2} \times \qquad (2.4)\\
& \times\left[\left(1-\frac{4m_\ell^2}{M_P^2}\right)\left|\frac{M_P^2(C_S-C'_S)}{2m_\ell(m_i+m_j)}\right|^2 + \left|\frac{M_P^2(C_P-C'_P)}{2m_\ell(m_i+m_j)} + C_A - C'_A\right|^2\right],
\end{aligned}$$

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where the Wilson coefficients are understood to be evaluated at a scale of $O(M_P^2)$. In $B_{s,d}$ modes this is the only relevant contribution to the branching ratios, whereas in K_L decays also long-distance effects have to be taken into account [9].

Within the SM only $\mathcal{O}_A$ has a non-vanishing Wilson coefficient,

$$C_A^{\rm SM}(M_W^2) = -V_{3i}^* V_{3j}\ Y_0\left(\frac{m_t^2}{M_W^2}\right) - V_{2i}^* V_{2j}\ Y_0\left(\frac{m_c^2}{M_W^2}\right), \tag{2.5}$$

where the charm contribution, which receives sizeable corrections in the running down to the meson scale, plays a non-negligible role only in the K_L case [10]. In the SUSY scenarios we are considering in this paper, all the C_i could have, in principle, a relevant impact on the decay amplitudes.

We stress that, since the ratios $(C_{S,P} - C'_{S,P})/m_\ell$ are independent of m_ℓ, to a first approximation the relative weight of the various contributions is independent of m_ℓ, whereas the overall branching ratios scale like m_ℓ^2. This implies that i) e^+e^- modes are never interesting; ii) the difficult detection of $\mathcal{B}(B_{s,d} \to \tau^+\tau^-)$ with respect to $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$ is partially compensated by an enhancement factor $\sim (m_\tau/m_\mu)^2$.

3. Axial operators

The contributions to the axial operators $\mathcal{O}_A$ and $\mathcal{O}'_A$ can be divided into two big categories: Z penguins and box diagrams. Those in the former class receive contributions from charginos, neutralinos and gluinos, while gluino exchange does not contribute to the latter class.

As discussed by several authors [11]–[14], box diagrams always play a subdominant role: in the case of $\tilde{q}_L^i$–$\tilde{q}_R^j$ mixing the Z penguins are by far dominant, in the case of $\mathrm{SU}(2)_L$-conserving mixing ($\tilde{q}_L^i$–$\tilde{q}_L^j$ or $\tilde{q}_R^i$–$\tilde{q}_R^j$) the constraints from $\Delta F = 2$ processes forbid sizeable effects.

Z-penguin diagrams correspond to dimension-six effective operators of the type $\bar{q}_{L(R)}^i \times \gamma^\mu q_{L(R)}^j H^\dagger D_\mu H$, where H denotes the Higgs field. For this reason, to be non-vanishing, these diagrams require a double chirality flip, which can arise either by the $\tilde{q}_L^i$–$\tilde{q}_R^j$ mixing or by the gaugino-Higgsino mixing (for charginos and neutralinos only). Also in this case, $\Delta F = 2$ constraints forbid sizeable effects in the case of $\mathrm{SU}(2)_L$-conserving squark mixing. As a consequence, possible large effects (with respect to the SM) are controlled only by the $\tilde{q}_L^i$–$\tilde{q}_R^j$ mixing terms, both in the up- and in the down-type sector.

3.1 Charginos

Involving up-type squarks, chargino contributions to Z penguins are mildly sensitive to the value of $\tan\beta$. These contributions, which we report here for completeness, have been widely discussed in the literature in the context of $B(K) \to X_s(X_d)\ell^+\ell^-$ processes [11]–[14]. Before any mass expansion, we can write [12]

$$[C_A]_\chi^{Z^0} = \frac{1}{8g^2} L_{nl}^j \bar{L}_{mk}^i F_{nmlk} \tag{3.1}$$

where

$$
\begin{aligned}
L^j_{nl} &= -g \hat{\Gamma}^U_{lj_L} \hat{V}^\dagger_{1n} + y^u_q V_{qj} \hat{\Gamma}^U_{lq_R} \hat{V}^\dagger_{2n} \,, \\
\bar{L}^i_{mk} &= -g (\hat{\Gamma}^U)^\dagger_{i_L k} \hat{V}_{m1} + y^u_q V^*_{qi} (\hat{\Gamma}^U)^\dagger_{q_R k} \hat{V}_{m2} \,, \\
F_{nmlk} &= \hat{V}_{n1} \hat{V}^\dagger_{1m} \delta_{lk} k(x_{mk}, x_{nk}) - 2 \hat{U}_{m1} \hat{U}^\dagger_{1n} \delta_{lk} \sqrt{x_{mk} x_{nk}}\, f(x_{mk}, x_{nk}) - \\
&\quad - \delta_{mn} \hat{\Gamma}^U_{kq_L} (\hat{\Gamma}^U)^\dagger_{q_L l} k(x_{mk}, x_{lk}) \,;
\end{aligned}
\tag{3.2}
$$

g is the $\mathrm{SU}(2)_L$ gauge coupling, V denotes the CKM matrix, $y^u_q = \sqrt{2} m_{u_q}/(v \sin\beta) = g m_{u_q}/(\sqrt{2} M_W \sin\beta)$ and the loop functions $k(x,y)$ and $f(x,y)$ can be found in the appendix.[1] Here $\hat{V}$ and $\hat{U}$ are the unitary matrices which diagonalize the chargino mass matrix ($\hat{U}^* M_\chi \hat{V}^\dagger = \mathrm{diag}(M_{\chi_1}, M_{\chi_2})$) and $\hat{\Gamma}^U$ is the one which diagonalizes the up-squark mass matrix, written in the basis where the $d^i_L - \tilde{u}^j_L - \chi_n$ coupling is family diagonal and the $d^i_L - \tilde{u}^j_R - \chi_n$ one is ruled by the CKM matrix.

The contribution of chargino Z-penguin diagrams to C'_A necessarily involves two down-type Yukawa couplings ($y^d_i y^d_j$), which cannot be both of the third family. As a result, even at large $\tan\beta$, this contribution is negligible with respect to the one in eq. (3.1).

A detailed discussion on the relative weight of the various terms appearing in eq. (3.1), up to the second order in the mass expansion, can be found in [12, 14]. Due to the off-diagonal CKM structure, the zeroth order term is not vanishing. However, this is not very interesting since it reaches at most $\mathcal{O}(10\%)$ of the SM contribution: the potentially largest effects are induced by terms of first or second order in the expansion of the squark mass matrix around the diagonal [12, 14]. A substantial simplification arises if, in addition to the squark mass matrix, also the chargino mass matrix is perturbatively expanded around the diagonal. In this case we can write

$$
\frac{[C_A]^{Z^0}_\chi}{C_A^{\mathrm{SM}}} \approx \frac{1}{8} \left[y_t \frac{(\delta^U_{RL})_{3j} \delta^\chi}{V_{3j}} f_1[x_{\alpha\beta}] + y_t \frac{(\delta^U_{LR})_{i3} \delta^\chi}{V^*_{3i}} f_1[x_{\alpha\beta}] + \frac{(\delta^U_{LR})_{i3} (\delta^U_{RL})_{3j}}{V^*_{3i} V_{3j}} f_2[x_{\alpha\beta}] \right] + \cdots \,,
\tag{3.3}
$$

where

$$
(\delta^U_{RL})_{ij} = \frac{(M^2_{\tilde{U}})_{i_R j_L}}{\langle M^2_{\tilde{q}} \rangle} \,, \qquad \delta^\chi = \frac{M_W}{\langle M_\chi \rangle} \,,
\tag{3.4}
$$

and the dots denote terms not enhanced by inverse CKM matrix elements, which can be safely neglected. The explicit expressions of the adimensional functions $f_{1,2}$, which depend on the various sparticle mass ratios (and mildly on $\tan\beta$), can be extracted from the full result in eq. (3.1): for $M_{\tilde{t}_R}/M_{\tilde{u}_L} \geq 1/2$ and $M_{\chi_n}/M_{\tilde{u}_L} \geq 1/3$, one finds $|f_1| \lesssim 0.1$ and $|f_2| \lesssim 0.4$ (the upper figures are obtained for the minimal value of $M_{\tilde{t}_R}$).

As anticipated, large effects are in principle possible in the presence of large $\tilde{u}^i_L$–$\tilde{u}^j_R$ mixing. However, both in the kaon and in the B system this possibility has recently been severely restricted by new experimental data on the non-helicity-suppressed modes. In the kaon case a clean and stringent bound is set by $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ [15]. Similarly, in the

[1] As usual $x_{\alpha\beta} = M^2_\alpha / M^2_\beta$, with the indices $(n,\, m)$ and $(l,\, k)$ denoting chargino and squark-mass eigenstates, respectively.

B system severe bounds are obtained by the recent data on both exclusive and inclusive $B_d \to X_s \ell^+ \ell^-$ transitions [16]. As a consequence of these new data, we are led to the model-independent conclusion that $|C_A/C_A^{\rm SM}| < 2$ in the $s \to d$ channel [17] and is even closer to unity in the $b \to s$ case [18]. The only channel without strong experimental bounds is the $b \to d$ one. However, once the vacuum-stability bounds of ref. [19] are imposed on $(\delta^U_{RL})_{31}$, also in this case one can at most obtain $|C_A/C_A^{\rm SM}| = O(1)$.

3.2 Gluinos

Contributions to Z penguins which involve down-type squarks, with gluinos and neutralinos, are very sensitive to the value of $\tan\beta$. In particular, at small $\tan\beta$ their effect is completely negligible because of the smallness of left-right down-type mass insertions. This conclusion is not necessarily true at large $\tan\beta$, where, in principle, left-right down-type mass insertions can become as large as up-type ones.

The largest effects are expected from gluino exchange, whose contribution to C_A, before any mass expansion, can be written as

$$[C_A]^{Z^0}_g = \frac{1}{8g^2} A^j_{bl} \bar{A}^i_{ak} I_{balk} \,, \tag{3.5}$$

where

$$\begin{aligned} A^j_{bl} &= -\sqrt{2} g_s T^b \hat{\Gamma}^D_{lj_L} \,, \\ \bar{A}^i_{ak} &= -\sqrt{2} g_s T^a (\hat{\Gamma}^D)^\dagger_{i_L k} \,, \\ I_{balk} &= \delta_{ab} \hat{\Gamma}^D_{kq_R} (\hat{\Gamma}^D)^\dagger_{q_R l} k(x_{gl}, x_{kl}) \,, \end{aligned} \tag{3.6}$$

g_s and T^a denote SU(3)$_c$ coupling and generators, and $\hat{\Gamma}^D$ is the matrix that diagonalizes the down-squark mass matrix (in the super-CKM basis). Contrary to the chargino exchange, in this case there is no zeroth order term in the mass expansion and C'_A is not negligible a priori: $[C'_A]^{Z^0}_g = [C_A]^{Z^0}_g (q_L \leftrightarrow q_R)$. However, the bounds on down-type mass insertions obtained from $\Delta F = 2$ processes are much stronger than in the up-type sector (because of the presence of $\Delta F = 2$ gluino-box amplitudes) [1]. Once these constraints (and the vacuum-stability ones) are taken into account, it is easy to realize that in all types of $d^i \to d^j$ transitions both $[C'_A]^{Z^0}_g$ and $[C_A]^{Z^0}_g$ can barely reach the magnitude of $|C_A^{\rm SM}|$. This conclusion is important since it ensures that the model-independent constraints on C_A discussed before cannot be invalidated by an accidental cancellation between C'_A and C_A.

The whole discussion on axial-vector operators can thus be summarized as follows: (i) at present, in all types of $d^i \to d^j$ transitions, axial-vector contributions can reach the SM level at most; (ii) the most efficient way to constrain (or detect) non-standard axial-vector contributions is by means of non-helicity-suppressed decay modes.

4. Scalar and pseudoscalar operators

As discussed in refs. [3]–[8], in the limit of large $\tan\beta$ Higgs-mediated scalar FCNCs can strongly modify, or even overcome, the SM amplitudes of rare helicity-suppressed modes.

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The origin of this effect is related to the appearance, at the one-loop level, of an effective coupling between H_U and down-type quarks [20]. As discussed in [4] (see also [22]), a necessary condition to take into account all $\tan\beta$-enhanced terms in FCNC amplitudes is the diagonalization of the down-type quark mass matrix in the presence of these effective couplings. Here we generalize this approach to the case where the effective $\bar{d}^i_R d^j_L H^*_U$ coupling contains also non-minimal sources of flavour mixing.[2]

Restricting the attention to the down-type neutral-Higgs sector, the effective down-type Yukawa interaction we need to consider is given by

$$\mathcal{L}^{\rm eff}_d = \bar{d}^i_R \left[y^d_i \delta_{ij} H^0_D + E_{ij} H^{0*}_U \right] d^j_L + \text{h.c.}\,, \tag{4.1}$$

where E_{ij} parametrizes the coupling generated at one loop by both chargino and gluino diagrams. Here we do not make any specific assumption about the flavour structure of E_{ij}, but require that the off-diagonal entries can be treated as perturbations with respect to the (full) diagonal mass terms in (4.1).[3] Then proceeding as in ref. [4] with a perturbative diagonalization of the mass terms, we find

$$\mathcal{L}^{\rm eff~(i>j)}_{\rm FCNC} = \frac{y^d_i \tan\beta}{\bar{y}^d_i} \left[E_{ij} + E^*_{ji} \left(\frac{\bar{y}^d_j}{\bar{y}^d_i} - \frac{y^d_j}{y^d_i} \right) \right] \left[\frac{1}{\tan\beta} H^{0*}_u - H^0_d \right] \bar{d}^i_R d^j_L + \text{h.c.}, \tag{4.2}$$

where

$$\bar{y}^d_i = y^d_i + E_{ii} \tan\beta\,. \tag{4.3}$$

The analogue of (4.2) for $i<j$ is obtained with the exchange $i \leftrightarrow j$ in all indices of Yukawa couplings (both y^d and $\bar{y}^d$), but not in the E_{ij} terms.

In the absence of non-minimal sources of flavour mixing, one has

$$E_{ij} = E^g_{ij} + E^\chi_{ij}\,, \qquad E^g_{ij} = \epsilon_g y^d_i \delta_{ij}, \qquad E^\chi_{ij} = \epsilon_Y y^d_i y^2_t V^{0*}_{3i} V^0_{3j}\,, \tag{4.4}$$

where V^0 denotes the bare CKM matrix (without the $\epsilon_Y \tan\beta$ corrections induced by re-diagonalization) and

$$\epsilon_g = \frac{2\alpha_s}{3\pi} \frac{\mu M_g}{M^2_{\tilde{d}_L}} f(x_{gd_L}, x_{d_R d_L})\,, \qquad \epsilon_Y = \frac{1}{16\pi^2} \frac{\mu A}{M^2_{\tilde{u}_L}} f(x_{\mu u_L}, x_{u_R u_L})\,. \tag{4.5}$$

Using eqs. (4.2)–(4.5), it is easy to recover the results of refs. [3, 4]. We shall now proceed with the evaluation of E^g_{ij} and E^χ_{ij} in the presence of non-minimal sources of flavour mixing.

[2]We stress that the so-called resummation of the $\tan\beta$-enhanced terms, in the limit where corrections of $O(M^2_W/\tilde{M}^2)$ are neglected, is nothing but the diagonalization of the effective Yukawa interaction, which includes the dimension-four effective operators appearing only at the quantum level [4, 22]. In particular, the computation of the coefficients of all relevant dimension-four effective operators to order $(\alpha_i/\pi)^1$, together with the diagonalization of the effective Yukawa interaction, leads to a resummation of all terms of order $(\alpha_i \tan\beta/\pi)^n$ in the limit $M^2_W \ll \tilde{M}^2$.

[3]In particular, this means we ignore the fine-tuned scenario where $E_{ii}\tan\beta \approx -y^d_i$.

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4.1 Charginos

The E_{ij} term induced by chargino–up-squark loops can be written as

$$E^{\chi}_{ij} = \frac{1}{16\pi^2} L^j_{nl} \bar{R}^i_{nk} G_{nlk} \,, \tag{4.6}$$

where

$$\bar{R}^i_{nk} = y^d_i (\hat{\Gamma}^U)^\dagger_{i_L k} \hat{U}^\dagger_{2n} \,, \tag{4.7}$$

$$G_{nlk} = y_t \hat{\Gamma}^U_{kq_L} (A_U V_0)^\dagger_{q_L t_R} (\hat{\Gamma}^U)^\dagger_{t_R l} \frac{M_{\chi_n}}{M^2_{\tilde{u}_k}} f(x_{lk}, x_{nk}) \,. \tag{4.8}$$

As usual, the trilinear soft-breaking term has been decomposed as $A_U \times Y_U$, and we have neglected all up-type Yukawa couplings but the top one. Since $E_{ij} \propto y^d_i$, only the case $i > j$ is relevant, and it is also clear that the E^*_{ji} term in eq. (4.2) can safely be neglected.

Integrating out the heavy neutral Higgs fields, in the large-$\tan\beta$ limit, we then obtain

$$[C_S]^{h^0,H^0,A^0}_{\chi} = [C_P]^{h^0,H^0,A^0}_{\chi} = \frac{\pi^2}{2G_F^2 M_W^2 M_A^2} \frac{y_\ell E^{\chi}_{ij} \tan\beta}{[1+\tan\beta(\epsilon_g + \epsilon_Y y_t^2 \delta_{i3})]} \,, \tag{4.9}$$

while the coefficients $[C'_{S,P}]^{h^0,H^0,A^0}_{\chi}$, suppressed by the factor y^d_j/y^d_i, turns out to be negligible. Due to the off-diagonal entries of the CKM matrix, even with a diagonal up-squark mass matrix $E^{\chi}_{i\neq j} \neq 0$. This zeroth order term leads to [4]:

$$\begin{aligned}
[C_S]^{h^0 H^0 A^0}_{\chi} &= [C_P]^{h^0 H^0 A^0}_{\chi} \\
&= \frac{m_{d_i} m_\ell m_t^2}{4M_W^2} \frac{\bar{\lambda}^t_{ij} \tan^3\beta}{[1+\tan\beta(\epsilon_g + \epsilon_Y y_t^2 \delta_{i3})]} \frac{\mu A f(x_{\mu u_L}, x_{u_R u_L})}{M^2_{\tilde{u}_L} M_A^2 [1+\epsilon_g \tan\beta]} \,,
\end{aligned} \tag{4.10}$$

where, expressing V^0 in terms of the physical CKM matrix,

$$\bar{\lambda}^t_{i>j} = \begin{cases} V^*_{33} V_{3j} & (i=3)\,, \\ V^*_{3i} V_{3j} \left[\frac{1+\tan\beta(\epsilon_g+\epsilon_Y y_t^2)}{1+\epsilon_g \tan\beta}\right]^2 & (i \neq 3)\,. \end{cases} \tag{4.11}$$

Interestingly, this zeroth order term in the mass-insertion approximation (MIA), turns out to be the most interesting contribution induced by chargino exchange. Indeed, the only possibility to remove the CKM suppression factor V_{3j} (the only one relevant in B decays) is by means of a double mass insertion: one in the chargino line, the other in the l-squark line. This leads to a potential enhancement factor $(\delta^U_{RL})_{3j}\delta^{\chi}/V_{3j}$, which is partially compensated by the suppression of the loop function with two mass insertions, i.e. a structure very similar to eq. (3.3). Then, taking into account the bounds on $(\delta^U_{RL})_{3j}$ discussed in section 3.1, it turns out that this contribution can be at most of the same order as the zeroth-order term, or the minimal-flavour-violating (MFV) term.

The suppression factor V^*_{3i}, which is relevant only to the kaon case, can be more easily removed: we simply need an off-diagonal trilinear term not proportional to V^*_{3i}. This leads to a net enhancement factor $(\delta^U_{LR})_{i3}/V^*_{3i}$, which could reach one order of magnitude. However, as discussed in ref. [4], the zeroth-order chargino contribution to the scalar $K_L \to \ell^+\ell^-$ amplitude amounts only to a few per cent of the SM one. Thus this potential enhancement is not particularly relevant.

4.2 Gluinos

The E_{ij} term induced by gluino–down-squark loops can be written as

$$E^g_{ij} = \frac{1}{16\pi^2} A^j_{bl} \bar{B}^i_{ak} J_{balk} \,, \tag{4.12}$$

where

$$\begin{aligned} \bar{B}^i_{ak} &= \bar{A}^i_{ak}(i_L \to i_R) \,, & (4.13) \\ J_{balk} &= \mu \delta_{ab} \hat{\Gamma}^D_{kq_R} y^d_q (\hat{\Gamma}^D)^\dagger_{q_L l} \frac{M_g}{M^2_{\tilde{d}_l}} f(x_{gd_l}, x_{d_k d_l}) \,. & (4.14) \end{aligned}$$

As can be noted, the gluino contribution to $E_{i\neq j}$ has a structure rather different from the chargino one. The latter is different from zero already at the lowest order in the MIA, but it is necessarily proportional to the Yukawa coupling y^d_i. On the other hand, $E^g_{i\neq j}$ vanishes at the zeroth order in the MIA; however, with a suitable mass insertion it can be made proportional to any of the down-type Yukawa couplings. As a result, in the gluino case also the $C'_{S,P}$ coefficients can play an important role.

Assuming for simplicity $i > j$, the leading contributions to the scalar operators induced by $E^g_{i\neq j}$ are given by

$$\begin{aligned} [C_S]^{h^0 H^0 A^0}_g = [C_P]^{h^0 H^0 A^0}_g &= \frac{4g_s^2}{3g^2} \frac{m_{d_i} m_\ell \mu M_{\tilde{g}}}{M^2_{A^0} M^2_{\tilde{d}}} \frac{\tan^3\beta}{[1+\tan\beta(\epsilon_g + \epsilon_Y y_t^2 \delta_{i3})]^2} \times \\ &\times \Big[(\delta^D_{LL})_{ij} x^2_{dd_L} f(x_{gd_L}, x_{d_R d_L}, 1) + & (4.15) \\ &\quad + \frac{y_b}{y^d_i} (\delta^D_{RR})_{i3} (\delta^D_{LL})_{3j} x^3_{dd_L} f(x_{gd_L}, x_{d_R d_L}, x_{d_R d_L}, 1) \Big] , \\ [C'_S]^{h^0 H^0 A^0}_g = -[C'_P]^{h^0 H^0 A^0}_g &= \frac{4g_s^2}{3g^2} \frac{m_{d_i} m_\ell \mu M_{\tilde{g}}}{M^2_{A^0} M^2_{\tilde{d}}} \frac{\tan^3\beta}{[1+\tan\beta(\epsilon_g + \epsilon_Y y_t^2 \delta_{i3})]^2} \times \\ &\times \Big[(\delta^D_{RR})_{ij} x^2_{dd_R} f(x_{gd_R}, x_{d_L d_R}, 1) + & (4.16) \\ &\quad + \frac{y_b}{y^d_i} (\delta^D_{LL})_{i3} (\delta^D_{RR})_{3j} x^3_{dd_R} f(x_{gd_R}, x_{d_L d_R}, x_{d_L d_R}, 1) \Big] , \end{aligned}$$

where the off-diagonal elements in the squark mass matrix have been parametrized in terms of flavour-changing insertions in the sfermion propagators, defined as in eq. (3.4). The terms arising at the second order in the MIA are relevant, and play a very important role, only in the $s \to d$ case $[(\delta^D_{LL(RR)})_{ii} \doteq 0]$. Indeed, similarly to what happens in chargino-induced Z penguins [12], only with a double flavour-mixing of the type $(2 \to 3) \times (3 \to 1)$ can the large Yukawa coupling of the third generation contribute to $s \to d$ amplitudes.

Before analysing the phenomenological implications of these formulae, we point out some differences with respect to previous analyses:

- Contrary to refs. [6, 7], the expressions (4.15) and (4.16) contain the leading (flavour-diagonal) $\epsilon \tan\beta$ corrections to all orders. As pointed out in [4], this is a numerically important contribution for $\tan\beta \sim 50$, where the largest effects are expected. In the limit where higher-order $\epsilon\tan\beta$ corrections are neglected, we find that $C^{g}_{S,P}$ is proportional to $\tan^3\beta$, similarly to $C^{\chi}_{S,P}$. This behaviour, which is explicit in ref. [6], is somehow hidden in eqs. (4.4)–(4.5) of ref. [7] (where the effective Yukawa interaction is not diagonalized). However, the $\tan^3\beta$ behaviour become manifest also in ref. [7] after an expansion of the $\hat{\Gamma}^D$ matrices: in this limit we fully agree with the overall normalization of ref. [7].[4]

- Within the limiting factors associated to the MIA, our formulae contain as a special case the scenario C of ref. [7]. In our opinion, such a scenario should not be denoted as a case of *minimal* flavour violation. Indeed diagonal — but flavour-non-universal — squark mass matrices do provide a *new* source of breaking of the $SU(3)^5$ flavour group [21, 22] and indeed they break the SM relation between Yukawa matrices and FCNC amplitudes. Since the purpose of this analysis is to identify the potentially largest effects generated by non-minimal sources of flavor mixing, and to identify the main differences with respect to MFV scenario (defined as in ref [22]), we prefer to treat all non-minimal cases in the same way, using the MIA. For the same reason, we shall not consider subleading terms such as the neutralino amplitude, which become important only in presence of cancellations among the leading terms [7].

5. Phenomenological bounds

As discussed in the previous two sections, only scalar (and pseudoscalar) operators can induce large enhancements of $B_{d,s} \to \ell^+\ell^-$ rates over the SM expectations. Among the contributions to these operators, the one induced by chargino exchange is likely to be dominated by the MFV term, widely discussed in the literature. On the other hand, the gluino contribution is extremely sensitive to possible new sources of flavour mixing. For this reason, in the following we shall concentrate only on gluino-mediated scalar amplitudes.

5.1 $B_{d,s} \to \mu^+\mu^-$

In the limit where $\tan\beta$ is large ($\tan\beta \gtrsim 30$), M_A is rather light ($M_A \lesssim 500\,\mathrm{GeV}$) and the off-diagonal mixing terms $(\delta^D_{LL(RR)})_{3i}$ are not vanishing, the gluino (scalar) amplitude could dominate both $B_d \to \ell^+\ell^-$ and $B_s \to \ell^+\ell^-$ rates. To get a rough idea of the possible effects, in the limit of degenerate sparticles we can write:

$$\frac{\mathcal{B}^g(B_d \to \mu^+\mu^-)}{\mathcal{B}^{\rm SM}(B_d \to \mu^+\mu^-)} \approx 3 \times 10^6 \left(\frac{200}{M_A}\right)^4 \left|(\delta^D_{LL(RR)})_{31}\right|^2 \frac{\left(\frac{\tan\beta}{50}\right)^6}{\left[\frac{2}{3} + \frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}, \tag{5.1}$$

$$\frac{\mathcal{B}^g(B_s \to \mu^+\mu^-)}{\mathcal{B}^{\rm SM}(B_s \to \mu^+\mu^-)} \approx 1.5 \times 10^5 \left(\frac{200}{M_A}\right)^4 \left|(\delta^D_{LL(RR)})_{32}\right|^2 \frac{\left(\frac{\tan\beta}{50}\right)^6}{\left[\frac{2}{3} + \frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}. \tag{5.2}$$

[4] This normalization differs from the one of ref. [6], which contains an extra factor of 4. We thank J. Urban for a clarifying discussion about this point.

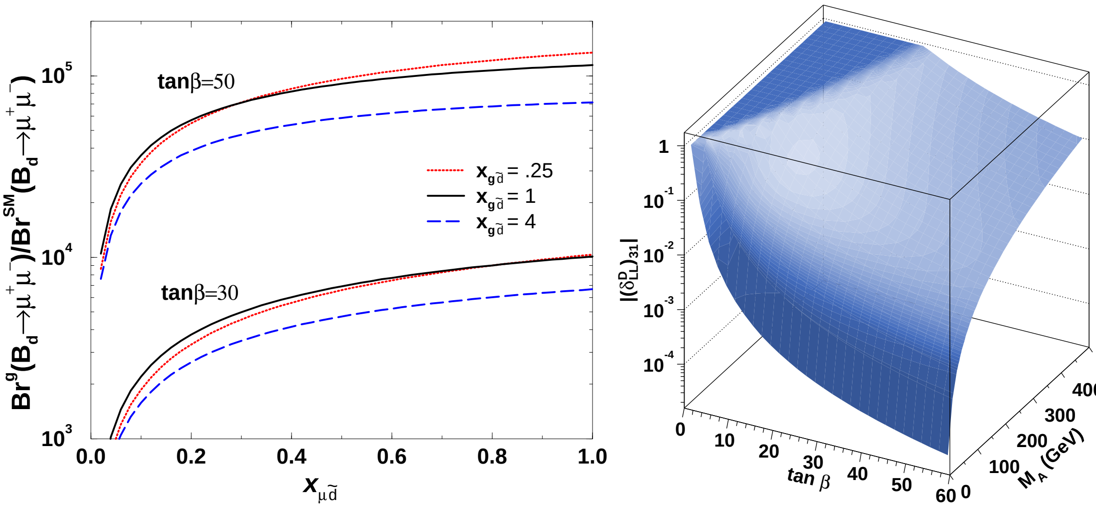

Figure 1: Left: $\mathcal{B}(B_d \to \mu^+\mu^-)$ normalized to the SM prediction as a function of $x_{\mu d_L}$, for $(\delta^D_{LL})_{31} = 0.1$, $M_A = 200$ GeV and different values of $\tan\beta$ and x_{gd_L}. Right: present bounds on $|(\delta^D_{LL(RR)})_{31}|$ imposed by the (scalar) gluino contribution to $\mathcal{B}(B_d \to \mu^+\mu^-)$ as a function of $\tan\beta$ and M_A, for $x_{\mu d_L} = x_{gd_L} = 1$.

These approximate numerical expressions hold at first order in the MIA, in the limit where we neglect any interference between supersymmetric and SM amplitudes and between supersymmetric contributions with different mass insertions. Moreover, we have neglected $\epsilon_Y y_t^2$ with respect to ϵ_g in the denominators of eqs. (4.15) and (4.16), and we have set ϵ_g to the value it assumes in the limit of degenerate supersymmetric scales, for $\mu > 0$ ($\epsilon_g \approx 0.012$).[5] A more complete illustration of the full dependence of $\mathcal{B}(B_d \to \mu^+\mu^-)$ from various supersymmetric parameters is provided by figure 1.

Using the present experimental bounds

$$\mathcal{B}(B_d \to \mu^+\mu^-) < 2.0 \times 10^{-7} \qquad 90\% \text{ C.L. [24]}, \tag{5.3}$$
$$\mathcal{B}(B_s \to \mu^+\mu^-) < 2.0 \times 10^{-6} \qquad 90\% \text{ C.L. [25]}, \tag{5.4}$$

we can derive a series of constraints on the squark flavour-mixing terms appearing in eqs. (4.15) and (4.16). These are reported in table 1 and illustrated in figure 1 (we show explicitly only the $1 \leftrightarrow 3$ case since the $2 \leftrightarrow 3$ is completely analogous, except for the rescaling of the SM contribution). All the bounds have been obtained in the limit of complete degeneracy among the eigenvalues of down-type squark masses. Moreover, we stress once more that these bounds are valid only in the limit where the gluino scalar amplitude corresponding to a given flavour-mixing term dominates completely the decay rate. If $(\delta^D_{LL})_{3i}$ and $(\delta^D_{RR})_{3i}$ are of the same order of magnitude, interference effects can no longer be neglected and leads to $O(1)$ modifications of these bounds. For instance for $(\delta^D_{LL})_{3i} = (\delta^D_{RR})_{3i}$ the bounds become more stringent by about a factor $\sqrt{2}$.

As already pointed out in ref. [6], for sufficiently large values of $\tan\beta$ the bounds on $|(\delta^D_{LL(RR)})_{31}|$ are more severe than those derived from the gluino-box contribution to B_d–$\bar{B}_d$

[5] The sign of μ is chosen according to the indications from $b \to s\gamma$ and $(g-2)_\mu$ [23].

		$\lvert(\delta^D_{LL(RR)})_{31}\rvert$		$\lvert(\delta^D_{LL(RR)})_{32}\rvert$	
$x_{\mu d_L}$	$x_{g d_L}$	$\tan\beta = 30$	$\tan\beta = 50$	$\tan\beta = 30$	$\tan\beta = 50$
1	1	0.089	0.026	0.22	0.065
0.25	1	0.13	0.035	0.33	0.086
0.25	4	0.16	0.043	0.39	0.11
		$\lvert\mathrm{Re}(\delta^D_{LL(RR)})_{21}\rvert$		$\lvert\mathrm{Re}[(\delta^D_{RR(LL)})_{23}(\delta^D_{LL(RR)})_{31}]\rvert$	
$x_{\mu d_L}$	$x_{g d_L}$	$\tan\beta = 30$	$\tan\beta = 50$	$\tan\beta = 30$	$\tan\beta = 50$
1	1	0.017	5.0×10^{-3}	0.7×10^{-3}	2.0×10^{-4}
0.25	1	0.025	6.6×10^{-3}	1.0×10^{-3}	2.7×10^{-4}
0.25	4	0.030	8.0×10^{-3}	1.4×10^{-3}	3.7×10^{-4}

Table 1: Present constraints on the off-diagonal squark mass terms that rule the (scalar) gluino contribution to $B_{s,d} \to \mu^+\mu^-$ (upper) and $K_L \to \mu^+\mu^-$ (lower). All the bounds have been obtained for $M_A = 200\,\mathrm{GeV}$ and scale as $(M_A/200\ \mathrm{GeV})^2$.

mixing [26]. However, we find that the effect is substantially reduced, with respect to what discussed in ref. [6], by the higher-order $\epsilon\tan\beta$ terms.

Already for $\tan\beta = 30$, the upper bounds obtained on $\lvert(\delta^D_{RR(LL)})_{32}\rvert$ are much more severe than those that could be derived, in the near future, from B_s–$\bar{B}_s$ mixing or the CP-violating asymmetry in $B_d \to \phi K_s$. As a result, if the scenario with a large $\tilde{b}_R$–$\tilde{s}_R$ mixing discussed in ref. [2] is accompanied by a large value of $\tan\beta$, which is a natural possibility in SO(10) models, the most interesting observables to look at are $\mathcal{B}\,(B_{s,d} \to \mu^+\mu^-)$. The clear signature of this scenario would be $\mathcal{B}(B_s \to \mu^+\mu^-)$ well above the SM expectation and, at the same time, a ratio $\mathcal{B}(B_d \to \mu^+\mu^-)/\mathcal{B}(B_s \to \mu^+\mu^-)$ different from $|V_{td}/V_{ts}|^2$ (i.e. the value expected in the absence of new flavour structures [22]).

5.2 $K_L \to \mu^+\mu^-$

Within the SM the short-distance contribution to the $K_L \to \mu^+\mu^-$ branching ratio is predicted to be $\mathcal{B}^{\mathrm{SM}}\,(K_L \to \mu^+\mu^-)_{\mathrm{SD}} = (8.7 \pm 3.7) \times 10^{-10}$ [10]. This expectation cannot be directly confronted with experimental data because of the long-distance amplitude generated by the two-photon intermediate state. Indeed the precise experimental measurement of $\mathcal{B}^{SM}\,(K_L \to \mu^+\mu^-)$ [27] turns out to be completely saturated by the absorptive two-photon contribution, signaling a cancellation between short-distance and dispersive long-distance amplitudes. Taking into account the estimates of the dispersive long-distance amplitude in refs. [9], we shall assume in the following the conservative upper bound

$$\mathcal{B}\,(K_L \to \mu^+\mu^-)_{\mathrm{SD}} < 3.0 \times 10^{-9}\,, \tag{5.5}$$

which should be regarded as a reference figure rather than as a strict bound.

As can be read from eqs. (4.15) and (4.16), gluino contributions to the $K_L \to \ell^+\ell^-$ amplitude suffer, at the first order in the MIA, by a double strong suppression: the smallness of the strange Yukawa coupling and the indirect constraints on $(\delta^D_{LL(RR)})_{21}$ [26]. However, the SM amplitude of $K_L \to \ell^+\ell^-$ decay is also affected by a strong suppression due to the CKM hierarchy $[\mathcal{A}(K_L \to \ell^+\ell^-)^{\mathrm{SM}} \sim \lambda^5]$. As a result, for large values of $\tan\beta$ this

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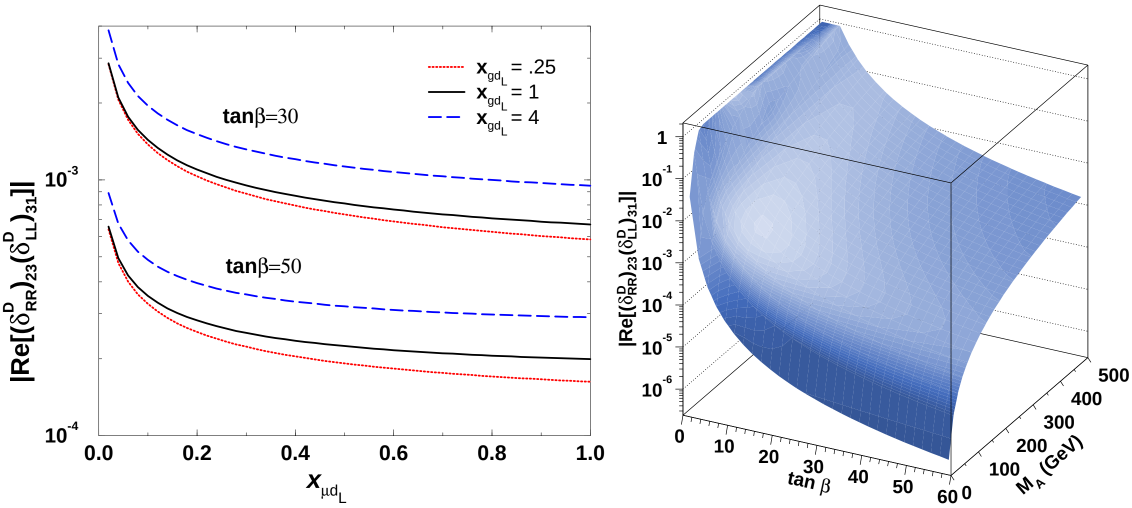

Figure 2: Constraints on $|\,\mathrm{Re}[(\delta^D_{RR(LL)})_{23}(\delta^D_{LL(RR)})_{31}]|$ set by $K_L \to \mu^+\mu^-$. Left: the bound as a function of $x_{\mu d_L}$, for $M_A = 200$ GeV and different values of $\tan\beta$ and x_{gd_L}. Right: the bound as a function of both $\tan\beta$ and M_A for $x_{\mu d_L} = x_{gd_L} = 1$.

supersymmetric amplitude could even become the dominant short-distance contribution. Adopting the same approximations as in eqs. (5.1) and (5.2), we can write

$$\frac{\mathcal{B}^{g(1)}(K_L \to \mu^+\mu^-)}{\mathcal{B}^{\rm SM}(K_L \to \mu^+\mu^-)_{\rm SD}} \approx \left(\frac{200}{M_A}\right)^4 \left|\frac{\mathrm{Re}(\delta^D_{LL(RR)})_{21}}{3\times 10^{-3}}\right|^2 \frac{\left(\frac{\tan\beta}{50}\right)^6}{\left[\frac{2}{3}+\frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}. \tag{5.6}$$

If $|\,\mathrm{Re}(\delta^D_{LL})_{12}| \gg |\,\mathrm{Im}(\delta^D_{LL})_{12}|$, as happens in many reasonable scenarios, the bound on $|\,\mathrm{Re}(\delta^D_{LL(RR)})_{21}|$ we can extract from this amplitude (see table 1) becomes competitive, for $\tan\beta \sim 50$, with those derived from the gluino-box contribution to K^0–$\bar{K}^0$ mixing [26].

As already emphasized, in rare K decays it is often necessary to go up to the second order in the MIA in order to take into account all possible large effects [12]. This occurs also in this case where, according to eqs. (4.15) and (4.16), at the second order in the MIA we can get rid of both suppression factors affecting the term with a single mass insertion. In the usual limit of degenerate supersymmetric scales, we can write

$$\frac{\mathcal{B}^{g(2)}(K_L \to \mu^+\mu^-)}{\mathcal{B}^{\rm SM}(K_L \to \mu^+\mu^-)_{\rm SD}} \approx \left(\frac{200}{M_A}\right)^4 \left|\frac{\mathrm{Re}[(\delta^D_{RR(LL)})_{23}(\delta^D_{LL(RR)})_{31}]}{1\times 10^{-4}}\right|^2 \frac{\left(\frac{\tan\beta}{50}\right)^6}{\left[\frac{2}{3}+\frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}. \tag{5.7}$$

Notice that the value of the effective coupling $|\,\mathrm{Re}[(\delta^D_{RR(LL)})_{23}(\delta^D_{LL(RR)})_{31}]|$ needed to reach the SM level is one order of magnitude smaller than the corresponding one required by $|\,\mathrm{Re}(\delta^D_{LL})_{12}|$. The precise bounds imposed by eq. (5.5) on the effective coupling $|\,\mathrm{Re}[(\delta^D_{RR(LL)})_{23}(\delta^D_{LL(RR)})_{31}]|$, for different values of supersymmetric parameters, are shown in figure 2 and reported in table 1. These bounds indicate that if $(\delta^D_{LL})_{ij}$ and $(\delta^D_{RR})_{ij}$ are of the same order, we cannot saturate at the same time the present limits on $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $\mathcal{B}(B_d \to \mu^+\mu^-)$. On the other hand, this possibility remains open if there exists a

		$\sqrt{\lvert(\delta^D_{RR})_{31}(\delta^D_{LL})_{31}\rvert}$		$\sqrt{\lvert(\delta^D_{RR})_{32}(\delta^D_{LL})_{32}\rvert}$	
$x_{\mu d_L}$	x_{gd_L}	$\tan\beta = 30$	$\tan\beta = 50$	$\tan\beta = 30$	$\tan\beta = 50$
1	1	0.005	2.3×10^{-3}	0.020	0.009
0.25	1	0.010	4.7×10^{-3}	0.039	0.019
0.25	4	0.011	5.4×10^{-3}	0.045	0.022

Table 2: Present constraints on the off-diagonal squark mass terms which contribute, via scalar amplitudes, to ΔM_{B_d} and ΔM_{B_s}. All the bounds have been obtained for $M_A = 200\,\mathrm{GeV}$ and scale linearly with M_A.

strong hierarchy between the two types of couplings. As we shall see, this conclusion is strongly reinforced by the constraints from B–$\bar{B}$ mixing.

5.3 B–$\bar{B}$ mixing

So far we compared the bounds on the mass insertions relevant to $\Delta F = 1$ scalar amplitudes, at large $\tan\beta$, with constraints imposed by $\Delta F = 2$ vector-type amplitudes (the gluino-box diagrams), which are independent from $\tan\beta$. However, as pointed out first in [28] (see also [29]), at large $\tan\beta$ scalar amplitudes could also play a significant role in $\Delta F = 2$ transitions. Indeed contracting the complex H_d field in eq. (4.2) with an appropriate FCNC coupling of H_d^* (hidden in the hermitean conjugate part of eq. (4.2)) we obtain $\Delta F = 2$ effective operators of the type $\bar{d}^i_R d^j_L \bar{d}^i_L d^j_R$ whose coupling scales like $\tan^4\beta$.

If E_{ij} is necessarily proportional to y^d_i, as happens in the chargino case, it is easy to realize that these $\Delta F = 2$ scalar operators are always suppressed by a small Yukawa coupling ($y^d_{j\neq 3}$) [28]. A similar situation occurs in the gluino case, if we allow only one type of mass insertion to be different from zero, i.e. $(\delta^D_{LL})_{3j}$ or $(\delta^D_{RR})_{3j}$. In these cases the only non-trivial constraints are set by B_s–$\bar{B}_s$ mixing, where this small Yukawa coupling is y_s. We have explicitly checked that these bounds are less stringent than those set by $\mathcal{B}(B_s \to \mu^+\mu^-)$, i.e. they do not exclude the possibility that $\mathcal{B}(B_s \to \mu^+\mu^-)$ is just below its present experimental limit.[6]

On the other hand, as we learned from the $P \to \ell^+\ell^-$ analysis, if we allow both LL and RR insertions to be different from zero we can have both E^g_{3j} and E^g_{j3} proportional to y_b and get rid of any small Yukawa coupling. In this case we obtain the following $\Delta F = 2$ effective hamiltonian:

$$\begin{aligned}
\mathcal{H}^{\mathrm{eff\,(scalar)}}_{\Delta B=2} &= -\frac{E^g_{3j}E^{g*}_{j3}\tan^2\beta}{M_A^2[1+\tan\beta(\epsilon_g+\epsilon_Y y_t^2)]^2}\,\bar{b}_R d^j_L \bar{b}_L d^j_R \\
&= -\frac{y_b^2\tan^2\beta(\delta^D_{LL})_{3j}(\delta^D_{RR})_{3j}}{[1+\tan\beta(\epsilon_g+\epsilon_Y y_t^2)]^2 M_A^2}\left(\frac{2\alpha_s\mu M_{\tilde{g}}}{3\pi M^2_{\tilde{d}}}\right)^2 \bar{b}_R d^j_L \bar{b}_L d^j_R \times \\
&\quad\times\left[x^2_{dd_L}x^2_{dd_R} f(x_{gd_L}, x_{d_R d_L}, 1) f(x_{gd_R}, x_{d_L d_R}, 1)\right], \qquad (5.8)
\end{aligned}$$

which have a large potential impact on both ΔM_{B_d} and ΔM_{B_s}. Employing the usual

[6] A similar conclusion was recently reported also in ref. [30].

approximation of degenerate supersymmetric scales, we can write

$$\left|\frac{\Delta M^{g}_{B_d}}{\Delta M^{\rm SM}_{B_d}}\right| \approx \left(\frac{200}{M_A}\right)^2 \left|\frac{(\delta^D_{LL})_{31}(\delta^D_{RR})_{31}}{6\times 10^{-6}}\right| \frac{\left(\frac{\tan\beta}{50}\right)^4}{\left[\frac{2}{3}+\frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}, \tag{5.9}$$

$$\left|\frac{\Delta M^{g}_{B_s}}{\Delta M^{\rm SM}_{B_s}}\right| \approx \left(\frac{200}{M_A}\right)^2 \left|\frac{(\delta^D_{LL})_{32}(\delta^D_{RR})_{32}}{1\times 10^{-4}}\right| \frac{\left(\frac{\tan\beta}{50}\right)^4}{\left[\frac{2}{3}+\frac{1}{3}\left(\frac{\tan\beta}{50}\right)\right]^4}, \tag{5.10}$$

The bounds obtained by the full analytical expressions, imposing the conditions $|\Delta M^{g}_{B_{d,s}}| < |\Delta M^{\rm SM}_{B_{d,s}}|$, are reported in table 2. The comparison between table 1 and table 2 clearly shows that down-type squark mixing could have a sizeable impact on $\mathcal{B}(B_{s,d}\to\mu^+\mu^-)$ modes only in the presence of a strong hierarchy between LL and RR insertions.

We finally mention that, in principle, one could extract bounds on the combination $(\delta^D_{LL})_{23}(\delta^D_{LL})_{31}(\delta^D_{RR})_{23}(\delta^D_{RR})_{31}$ from K–$\bar{K}$ mixing. We have explicitly checked that the bounds imposed by ΔM_K on $\sqrt{|\,{\rm Re}[(\delta^D_{LL})_{23}(\delta^D_{LL})_{31}(\delta^D_{RR})_{23}(\delta^D_{RR})_{31}]|}$ are slightly less stringent than those obtained from $K_L\to\mu^+\mu^-$ on $|\,{\rm Re}[(\delta^D_{LL})_{23}(\delta^D_{RR})_{31}]|$ and reported in table 1. This happens because, contrary to $B_{s,d}\to\mu^+\mu^-$ modes, we can allow at most $O(1)$ effects in $K_L\to\mu^+\mu^-$.

6. Conclusions

The rare dilepton decays $B_{s,d}\to\ell^+\ell^-$ offer a unique opportunity to study the scalar FCNCs which naturally arise in supersymmetry at large $\tan\beta$. In this paper we have complemented previous studies of these decays modes in supersymmetry [3]–[8] analysing their sensitivity to non-minimal sources of flavour mixing.

Similarly to the MFV scenario [3, 4], we have shown that also in this case it is necessary to diagonalize the effective Yukawa interaction — including the $\bar{d}^i_R d^j_L H^*_U$ coupling generated by the new flavour structures — in order to identify the correct $\tan\beta$ behaviour of scalar FCNCs and to take into account all the leading $\tan\beta$-enhanced terms. Using this approach, we have corrected/completed previous results on $\mathcal{A}(B_{s,d}\to\ell^+\ell^-)$ in the presence of non-minimal sources of flavour mixing [6, 7].

An important difference with respect to the MFV scenario arises because the effective $\bar{d}^i_R d^j_L H^*_U$ operator generated by gluino-squark loops is not necessarily proportional to the Yukawa coupling of the right-handed external quark (y^d_i): allowing both left-left and right-right mass insertions to be different from zero, we can generate potentially large contributions suppressed only by the bottom Yukawa coupling (even for both $i\neq 3$ and $j\neq 3$). This leads to extract stringent bounds on the flavour off-diagonal down-type squark mixing, at large $\tan\beta$, from $\Delta M_{B_{d,s}}$ and $K_L\to\mu^+\mu^-$. These bounds, which have been analysed for the first time as reported in this paper, are summarized in tables 1 and 2.

As a result of the bounds from $\Delta M_{B_{d,s}}$ and $K_L\to\mu^+\mu^-$, we reached the conclusion that flavour off-diagonal squark mixing can induce order-of-magnitude enhancements of $\mathcal{B}(B_{s,d}\to\ell^+\ell^-)$ only in the presence of a strong hierarchy between left-left and right-right flavour-mixing terms. This scenario, which is apparently fine-tuned, could be generated in a natural way in GUT models, as pointed out for instance in ref. [2]. If this is the case, one should expect a ratio $\mathcal{B}(B_d \to \mu^+\mu^-)/\mathcal{B}(B_s \to \mu^+\mu^-)$ very different from $|V_{td}/V_{ts}|^2$, i.e. the value expected in the absence of new flavour structures.

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A. Loop functions

The loop functions appearing in the chargino-squark and gluino-squark penguin amplitudes discussed in sections 3 and 4, can be derived from the recursive formula [11, 12]

$$I(x,y,z_1,\ldots,z_{n-2}) = \frac{I(x,z_1,\ldots,z_{n-2}) - I(y,z_1,\ldots,z_{n-2})}{x-y}, \tag{A.1}$$

starting from the single variable functions

$$f(x) = \frac{x\log x}{x-1} \qquad \text{and} \qquad k(x) = x f(x)\,. \tag{A.2}$$

Some useful reference values are

$$f(1,1) = \frac{1}{2}, \qquad f(1,1,1) = -\frac{1}{6}, \qquad f(1,1,1,1) = \frac{1}{12}. \tag{A.3}$$

References

[1] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of fcnc and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387];
A. Masiero and O. Vives, *New physics in CP-violation experiments*, *Ann. Rev. Nucl. Part. Sci.* **51** (2001) 161 [hep-ph/0104027].

[2] T. Moroi, *CP-violation in $B_d \to \phi K_S$ in SUSY GUT with right-handed neutrinos*, *Phys. Lett.* **B 493** (2000) 366 [hep-ph/0007328];
D. Chang, A. Masiero and H. Murayama, *Neutrino mixing and large CP-violation in B physics*, hep-ph/0205111.

[3] C. Hamzaoui, M. Pospelov and M. Toharia, *Higgs-mediated FCNC in supersymmetric models with large* $\tan\beta$, *Phys. Rev.* **D 59** (1999) 095005 [hep-ph/9807350];
K.S. Babu and C.F. Kolda, *Higgs-mediated $B^0 \to \mu^+\mu^-$ in minimal supersymmetry*, *Phys. Rev. Lett.* **84** (2000) 228 [hep-ph/9909476].

[4] G. Isidori and A. Retico, *Scalar flavour-changing neutral currents in the large-*$\tan\beta$ *limit*, *J. High Energy Phys.* **11** (2001) 001 [hep-ph/0110121].

[5] C.-S. Huang, W. Liao and Q.-S. Yan, *The promising process to distinguish supersymmetric models with large* $\tan\beta$ *from the standard model: $B \to X_s\mu^+\mu^-$*, *Phys. Rev.* **D 59** (1999) 011701 [hep-ph/9803460];

JHEP09(2002)063

S.R. Choudhury and N. Gaur, *Dileptonic decay of B_s meson in SUSY models with large* $\tan\beta$, *Phys. Lett.* **B 451** (1999) 86 [hep-ph/9810307];
C.-S. Huang, W. Liao, Q.-S. Yan and S.-H. Zhu, *$B_s \to \ell^+\ell^-$ in a general 2HDM and MSSM*, *Phys. Rev.* **D 63** (2001) 114021 [hep-ph/0006250];
C. Bobeth, T. Ewerth, F. Kruger and J. Urban, *Analysis of neutral Higgs-boson contributions to the decays $\bar{B}_s \to \ell^+\ell^-$ and $\bar{B} \to K\ell^+\ell^-$*, *Phys. Rev.* **D 64** (2001) 074014 [hep-ph/0104284];
Z. Xiong and J.M. Yang, *Rare b-meson dileptonic decays in minimal supersymmetric model*, *Nucl. Phys.* **B 628** (2002) 193 [hep-ph/0105260].

[6] P.H. Chankowski and L. Slawianowska, *$B^0_{d,s} \to \mu^-\mu^+$ decay in the MSSM*, *Phys. Rev.* **D 63** (2001) 054012 [hep-ph/0008046].

[7] C. Bobeth, T. Ewerth, F. Kruger and J. Urban, *Enhancement of $\mathcal{B}(\bar{B}_d \to \mu^+\mu^-)/\mathcal{B}(\bar{B}_s \to \mu^+\mu^-)$ in the MSSM with minimal flavour violation and large tan beta*, hep-ph/0204225.

[8] A. Dedes, H.K. Dreiner and U. Nierste, *Correlation of $B_s \to \mu^+\mu^-$ and $(g-2)_\mu$ in minimal supergravity*, *Phys. Rev. Lett.* **87** (2001) 251804 [hep-ph/0108037];
S.-W. Baek, P. Ko and W.Y. Song, *Implications on SUSY breaking mediation mechanisms from observing $B_s \to \mu^+\mu^-$ and the muon $(g-2)$*, hep-ph/0205259; *SUSY breaking mediation mechanisms and $(g-2)_\mu$, $B \to X_s\ell^+\ell^-$ and $B_s \to \mu^+\mu^-$*, hep-ph/0208112;
A. Dedes, H.K. Dreiner, U. Nierste and P. Richardson, *Trilepton events and $B_s \to \mu^+\mu^-$: no-lose for msugra at the Tevatron?*, hep-ph/0207026;
J.K. Mizukoshi, X. Tata and Y. Wang, *Higgs-mediated leptonic decays of B_s and B_d mesons as probes of supersymmetry*, hep-ph/0208078.

[9] G. D'Ambrosio, G. Isidori and J. Portoles, *Can we extract short-distance information from $B(K_L \to \mu^+\mu^-)$?*, *Phys. Lett.* **B 423** (1998) 385 [hep-ph/9708326];
G. Valencia, *Long distance contribution to $K_L \to \ell^+\ell^-$*, *Nucl. Phys.* **B 517** (1998) 339 [hep-ph/9711377];
D. Gomez Dumm and A. Pich, *Long-distance contributions to the $K_L \to \mu^+\mu^-$ decay width*, *Phys. Rev. Lett.* **80** (1998) 4633 [hep-ph/9801298].

[10] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380].

[11] Y. Nir and M.P. Worah, *Probing the flavor and CP structure of supersymmetric models with $K \to \pi\nu\bar{\nu}$ decays*, *Phys. Lett.* **B 423** (1998) 319 [hep-ph/9711215];
A.J. Buras, A. Romanino and L. Silvestrini, *$K \to \pi\nu\bar{\nu}$: a model independent analysis and supersymmetry*, *Nucl. Phys.* **B 520** (1998) 3 [hep-ph/9712398].

[12] G. Colangelo and G. Isidori, *Supersymmetric contributions to rare kaon decays: beyond the single mass-insertion approximation*, *J. High Energy Phys.* **09** (1998) 009 [hep-ph/9808487].

[13] A.J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, *Connections between ϵ'/ϵ and rare kaon decays in supersymmetry*, *Nucl. Phys.* **B 566** (2000) 3 [hep-ph/9908371].

[14] E. Lunghi, A. Masiero, I. Scimemi and L. Silvestrini, *$B \to X_s\ell^+\ell^-$ decays in supersymmetry*, *Nucl. Phys.* **B 568** (2000) 120 [hep-ph/9906286].

[15] E787 collaboration, S. Adler et al., *Further evidence for the decay $K^+ \to \pi^+\nu\bar{\nu}$*, *Phys. Rev. Lett.* **88** (2002) 041803 [hep-ex/0111091]; *Further search for the decay $k^+ \to \pi^+\nu\bar{\nu}$*, *Phys. Rev. Lett.* **84** (2000) 3768 [hep-ex/0002015]; *Evidence for the decay $K^+ \to \pi^+\nu\bar{\nu}$*, *Phys. Rev. Lett.* **79** (1997) 2204 [hep-ex/9708031].

[16] BELLE collaboration, K. Senyo, *$B \to X_s \ell^+ \ell^-$ at Belle*, hep-ex/0207005;
BELLE collaboration, K. Abe et al., *Observation of the decay $B \to K \mu^+ \mu^-$*, *Phys. Rev. Lett.* **88** (2002) 021801 [hep-ex/0109026];
BABAR collaboration, B. Aubert et al., *Search for the rare decays $B \to K \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$*, *Phys. Rev. Lett.* **88** (2002) 241801 [hep-ex/0201008].

[17] G. D'Ambrosio and G. Isidori, *$K^+ \to \pi^+ \nu \bar{\nu}$: a rising star on the stage of flavour physics*, *Phys. Lett.* **B 530** (2002) 108 [hep-ph/0112135].

[18] A. Ali, E. Lunghi, C. Greub and G. Hiller, *Improved model-independent analysis of semileptonic and radiative rare B decays*, *Phys. Rev.* **D 66** (2002) 034002 [hep-ph/0112300].

[19] J.A. Casas and S. Dimopoulos, *Stability bounds on flavor violating trilinear soft terms in the MSSM*, *Phys. Lett.* **B 387** (1996) 107 [hep-ph/9606237].

[20] L.J. Hall, R. Rattazzi and U. Sarid, *The top quark mass in supersymmetric* SO(10) *unification*, *Phys. Rev.* **D 50** (1994) 7048 [hep-ph/9306309].

[21] R.S. Chivukula and H. Georgi, *Composite technicolor standard model*, *Phys. Lett.* **B 188** (1987) 99.

[22] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, hep-ph/0207036.

[23] See S.P. Martin and J.D. Wells, *Muon anomalous magnetic dipole moment in supersymmetric theories*, *Phys. Rev.* **D 64** (2001) 035003 [hep-ph/0103067] and references therein.

[24] BABAR collaboration, B. Aubert et al., *Search for decays of B^0 mesons into pairs of leptons*, hep-ex/0207083.

[25] CDF collaboration, F. Abe et al., *Search for the decays $B_d^0 \to \mu^+ \mu^-$ and $B_s^0 \to \mu^+ \mu^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV*, *Phys. Rev.* **D 57** (1998) 3811.

[26] M. Ciuchini et al., *ΔM_K and ϵ_K in SUSY at the next-to-leading order*, *J. High Energy Phys.* **10** (1998) 008 [hep-ph/9808328].

[27] E871 collaboration, D. Ambrose et al., *Improved branching ratio measurement for the decay $K_L^0 \to \mu^+ \mu^-$*, *Phys. Rev. Lett.* **84** (2000) 1389.

[28] A.J. Buras, P.H. Chankowski, J. Rosiek and L. Slawianowska, *$\Delta M_s / \Delta M_d$, $\sin 2\beta$ and the angle γ in the presence of new $\Delta F = 2$ operators*, *Nucl. Phys.* **B 619** (2001) 434 [hep-ph/0107048].

[29] A.J. Buras, P.H. Chankowski, J. Rosiek and L. Slawianowska, *Correlation between ΔM_s and $B_{s,d}^0 \to \mu^+ \mu^-$ in supersymmetry at large $\tan\beta$*, hep-ph/0207241.

[30] P.H. Chankowski and J. Rosiek, *Supersymmetry (at large $\tan\beta$) and flavor physics*, hep-ph/0207242.